Section 5.3 – Extra Practice – Solving Trig Equations

Solve the following trig equations and give all possible solutions in the interval $[0, 2\pi)$.

1. $\sin^2\theta = \cos^2\theta$

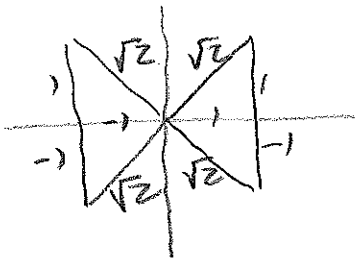
$\sin^2\theta = 1 - \sin^2\theta$

$2\sin^2\theta = 1$

$\sin^2\theta = \frac{1}{2}$

$\sin\theta = \pm\sqrt{\frac{1}{2}} = \pm\frac{1}{\sqrt{2}}$

$\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$



2. $3\cot^2\theta - 1 = 0$

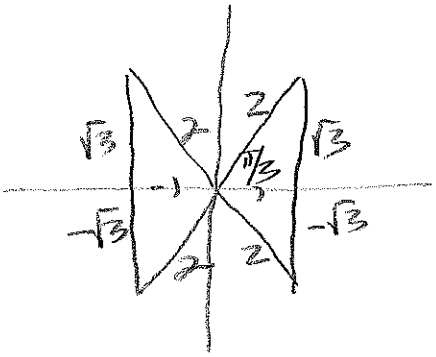
$3\cot^2\theta = 1$

$\cot^2\theta = \frac{1}{3}$

$\cot\theta = \pm\sqrt{\frac{1}{3}} = \pm\frac{1}{\sqrt{3}}$

$\Rightarrow \tan\theta = \pm\sqrt{3}$

$\theta = \pi/3, 2\pi/3, 4\pi/3, 5\pi/3$



4. $2\sin^2 x - 3\sin x = (1) - 1$

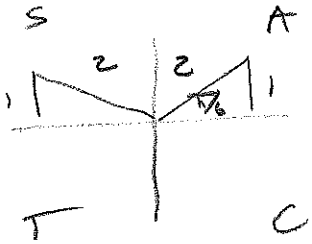
$2\sin^2 x - 3\sin x + 1 = 0$

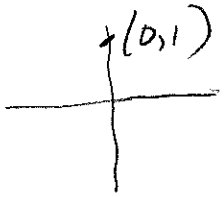
$(2\sin x - 1)(\sin x - 1) = 0$

$2\sin x - 1 = 0$ or $\sin x - 1 = 0$

$\sin x = \frac{1}{2}$ or $\sin x = 1$

$x = \pi/6, 5\pi/6$ or $x = \pi/2$





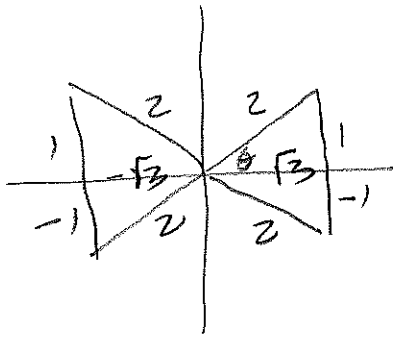
Section 5.3 – Extra Practice – Solving Trig Equations

5. $3\sec^2 x - 4 = 0$

$\sec^2 x = \frac{4}{3}$

$\sec x = \pm\sqrt{\frac{4}{3}} = \pm\frac{2}{\sqrt{3}}$

$\Rightarrow \cos x = \pm\frac{\sqrt{3}}{2}$



$\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

6. $3\tan^3\theta = \tan\theta$

$3\tan^3\theta - \tan\theta = 0$

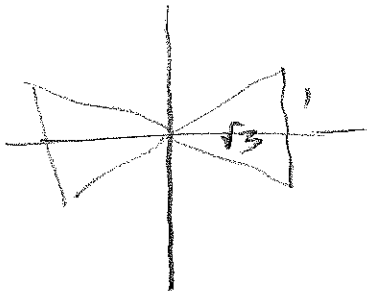
$\tan\theta(3\tan^2\theta - 1) = 0$

$\tan\theta = 0$ so $\sin\theta = 0$

$3\tan^2\theta - 1 = 0$

$\tan^2\theta = \frac{1}{3}$

$\tan\theta = \pm\sqrt{\frac{1}{3}} = \pm\frac{1}{\sqrt{3}}$



$\theta = 0, \pi$ $\quad \theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

7. $4\cos\theta = 1 + 2\cos\theta$

$4\cos\theta - 2\cos\theta = 1$

$2\cos\theta = 1$

$\cos\theta = \frac{1}{2}$

$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$